ELEMENTARY LINEAR ALGEBRA – SET 8

Eigenvalues and eigenvectors

1. Determine the (real) eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Find the (complex) eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 1 & -5 \\ 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 8 & 0 \\ 0 & 0 & -2 \\ 2 & 8 & -2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

3. Find the eigenvalues and eigenvectors of the following linear mappings:
(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, where $T(x, y) = (x + 2y, x - y)$
(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$, where $T(x, y, z) = (y, x, z)$
(c) $T : \mathbb{C}^3 \to \mathbb{C}^3$, where $T(x, y, z) = (x + 2y + z, -2x + y, z)$

4. Let T be the reflection of the space $\mathbb{R}^2$ with respect to the x axis. Using the geometric interpretation of T, determine its eigenvalues and eigenvectors.

5. Diagonalize the real matrices

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 4 & 0 & 6 \\ 2 & 1 & 4 \\ -1 & 0 & -1 \end{pmatrix}$$

6. Check whether the following matrix is diagonalizable:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

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